Integration

Indefinite Integral (No limits, Constant of integration): $\int \mathrm{f}(x)\,\mathrm{d}x = \mathrm{g}(x) + c.$

f(x) is the Integrand

Definite Integral (Limits):

$$\int_a^b \mathrm{f}(x)\,\mathrm{d}x = \left[\mathrm{g}(x)\right]_a^b = \mathrm{g}(b) - \mathrm{g}(a)$$

1. $\int x^n\,\mathrm{d}x = \dfrac{x^{n+1}}{n+1} + c, \qquad n \neq -1.$
2. $\int k\,\mathrm{d}x = kx + c, \qquad (k \text{ is a constant}).$
3. $\int (ax+b)^n\,\mathrm{d}x = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c, \qquad (a \text{ \& } b \text{ are constants \& } n \neq -1).$
4. $\int x^{-1}\,\mathrm{d}x = \ln|x| + c.$
5. $\int (ax+b)^{-1}\,\mathrm{d}x = \int \dfrac{1}{ax+b}\,\mathrm{d}x = \dfrac{\ln|ax+b|}{a} + c.$
6. Fractions: (a) $\dfrac{\mathrm{f}'(x)}{\mathrm{f}(x)}$, then $\int \dfrac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c.$

   (b) Improper fraction $\dfrac{\mathrm{g}(x)}{\mathrm{h}(x)}$: $\div\, \mathrm{g}(x)$ by $\mathrm{h}(x)$. Rewrite as $\text{Quotient} + \dfrac{\text{Remainder}}{\text{Divisor}}$, then integrate.

   (c) If $\mathrm{h}(x)$ is of the form

   (i) $(ax+b)(cx+d)(ex+f)\ldots$
   (ii) $(ax+b)^2$
   (iii) ax^2+bx+c that cannot be factorised

   } Express into partial fractions before integrating

7. Trigonometric: (a) $\int \sin(ax+b)\,\mathrm{d}x = -\dfrac{\cos(ax+b)}{a} + c$

   $\int \cos(ax+b)\,\mathrm{d}x = \dfrac{\sin(ax+b)}{a} + c$

   $\int \sec^2(ax+b)\,\mathrm{d}x = \dfrac{\tan(ax+b)}{a} + c$

   (b) $\int \tan(ax+b)\,\mathrm{d}x = -\dfrac{\ln|\cos(ax+b)|}{a} + c$

   $\int \cot(ax+b)\,\mathrm{d}x = \dfrac{\ln|\sin(ax+b)|}{a} + c$

   (c) $\sin^2$, use identity $\cos 2x \equiv 1 - 2\sin^2 x$ & express integrand in terms of $\cos 2x$.
   $\cos^2$, use identity $\cos 2x \equiv 2\cos^2 x - 1$ & express integrand in terms of $\cos 2x$.
   $\tan^2$, use identity $1 + \tan^2 x \equiv \sec^2 x$ & express integrand in terms of $\sec^2$.

8. Exponential: $\int e^{ax+b}\,\mathrm{d}x = \dfrac{e^{ax+b}}{a} + c.$
9. Product of $\mathrm{g}(x)\mathrm{h}(x)$. Integration by parts, $\int u\mathrm{d}v\,\mathrm{d}x = uv - \int v\mathrm{d}u\,\mathrm{d}x$. Use the acronym LATE to decide whether $\mathrm{g}(x)$ or $\mathrm{h}(x)$ will be u & dv.

Substitution technique. (1) Differentiate the given substitution $u = \mathrm{g}(x)$ to find $\dfrac{\mathrm{d}u}{\mathrm{d}x}$. Express $\mathrm{d}x$ in terms u.

(2) Change the integrand in terms of u.

(3) For Indefinite integral: Integrate wrt u & express the answer in terms of x.

For Definite integral: Change the limits for x by using the substitution to find values for u.

Integrate wrt u