ROUNDING

Rounding means reducing the digits in a number while trying to keep its value similar.

How to Round Numbers

1. Decide which is the last digit to keep
2. Leave it the same if the next digit is less than 5 (this is called rounding down)
3. But increase it by 1 if the next digit is 5 or more (this is called rounding up)

Example 1: Round 83.2 to the nearest whole number.

To round to the nearest whole number means that there should be no digits after the decimal point. Consequently we have to stop with the digit 3 (Step 1). The next digit is 2 which is less than 5. Therefore, we round down by leaving 3 unchanged (Step 2) and the answer is 83.

Example 2: Round 86.78 kg to the nearest kg.

Rounding to the nearest kg (meter, liter or any unit) means rounding to the nearest whole number. We have to stop with the digit 6 (Step 1). The next digit is 7 which is more than 5. Therefore we round up by increasing 6 by 1 (Step 3) and the answer is 87.

Example 3: Round the following to the nearest whole number.

a) 6.5 becomes 7. (Rounding up because of 5 after 6)
b) 102.85 becomes 103. (Rounding up because of 8 after 2)
c) 1000.153 becomes 1000. (Rounding down because of 1 after 0)
d) 209.6 becomes 210. (Rounding up because of 6 after 9, adding 1 to 209)
e) 99.92 becomes 100. (Rounding up because of 9 after 9, adding 1 to 99)

DECIMAL POINT

A decimal mark is a symbol used to separate the integer part from the fractional part of a number written in decimal form. Different countries officially designate different symbols for the decimal mark. The 22nd General Conference on Weights and Measures declared in 2003 that "the symbol for the decimal marker shall be either the point on the line or the comma on the line". Source (http://en.wikipedia.org/wiki/Decimal_mark)

The number of 'decimal places' equals to the number of digits after the decimal point. For example, 2.31 has 2 decimal places, 10.0 has 1 decimal place)

How to Round Decimals

Same steps as for rounding numbers.

Example 1: Round 3.141592654 to

a) 2 decimal places
We need only 2 digits after the decimal point, that is, we stop with 4. The next digit is 1 which is less than 5. Therefore we round down and the answer is 3.14.
b) 3 decimal places
This time, we need 3 digits after the decimal point, that is, we stop with 1. The next digit is 5 which is equal to 5. Therefore we round up by adding 1 to 1 and the answer is 3.142.
c) 4 decimal places
Now, we need 4 digits after the decimal point that is we stop with 5. The next digit is 9 which is more than 5. Therefore we round up by adding 1 to 5 and the answer is 3.1416.

Example 2: Write 213.04601 cm to the nearest 0.1 cm

To the nearest 0.1 cm means 1 decimal place (0.01 would mean 2 decimal places). We need to stop with 0. The next digit is 4 which is less than 5. Therefore we round down and the answer is 213.0 cm.

Example 3: Write 97.268910 kg to the nearest 0.001 kg

To the nearest 0.001 kg means to 3 decimal places because there are 3 digits after the decimal point in 0.001. Hence we stop with 8. The next digit is 9. Consequently we round up and the answer is 97.269.

SIGNIFICANT FIGURES

The significant figures of a number are those digits that carry meaning, contributing to its precision. Specifically, the rules for identifying significant figures when writing or interpreting numbers are as follows:

1. All non-zero digits are considered significant. For example, 91 has two significant figures (9 and 1), while 123.45 has five significant figures (1, 2, 3, 4 and 5).
2. Zeros appearing anywhere between two non-zero digits are significant. Example: 101.1203 has seven significant figures: 1, 0, 1, 1, 2, 0 and 3.
3. Leading zeros are not significant. For example, 0.00052 has two significant figures: 5 and 2.
4. Trailing zeros in a number containing a decimal point are significant. For example, 12.2300 has six significant figures: 1, 2, 2, 3, 0 and 0. The number 0.000122300 still has only six significant figures (the zeros before the 1 are not significant). In addition, 120.00 has five significant figures since it has three trailing zeros.
5. The significance of trailing zeros in a number not containing a decimal point can be ambiguous. For example, it may not always be clear if a number like 1300 is precise to the nearest unit (and just happens coincidentally to be an exact multiple of a hundred) or if it is only shown to the nearest hundred due to rounding or uncertainty.

Source (http://en.wikipedia.org/wiki/Significant_figures)

How to round off to *n* significant figures

Rounding up and rounding down still applies but we have to be careful with which digits are significant or not.

Examples: Write the following to the number of significant figures indicated between brackets.

a) 78594 (3 s.f)
The 3rd significant digit in 78594 is 5. The next digit is 9. Therefore, we round up by increasing 5 by 1 and replacing 9 and 4 with 0. The answer is 78600
If we write 78594 to 2 s.f, the answer will be 79000 because the 2nd significant digit is 8 and the next digit is 5. Therefore we round up and replace all digits after 8 by 0.
If we write 78594 to 4 s.f, the answer is 78590 because the 4th significant digit is 9 and the next digit is 4. We round down and replace the digits after 9 by 0.
Finally, 78594 to 1 s.f becomes 80000 because the 1st significant figure is 7 and the next digit is 8. So we round up and replace digits after 7 by 0.
b) 130.681 (4 s.f)
The 0 in this number is significant because it appears between 2 non-zero digits. Consequently, the 4th significant digit is 6 and the next digit is 8. We round up and the answer is 130.7. We should not replace the digits after the significant digit by 0 because they are in the decimal part (.681) of the number.
If we write 130.681 to 3 s.f, the answer is 131 because the 3rd significant digit is 0 and the next digit is 6. We round up.
If we write 130.681 to 2 s.f, the answer is 130 because the 2nd significant digit is 3 and next digit is 0. We round down and have to replace digits in the integer part (130) only by 0 (0 is replaced by 0 in this case).
c) 0.0053276 (1 s.f)
The 0 before 5 are called leading zeros and they are insignificant. Therefore the 1st significant digit is 5 and the next digit is 3. We round down and the answer is 0.005.
If we write 0.0053276 to 3 s.f, the answer is 0.00533 because the 3rd significant digit is 2 and the next digit is 7. So we round up.
d) 1.073600 (2 s.f)
All the 0 in this number are significant. So the 2nd significant digit is 0 and the next digit is 7. We round up and the answer is 1.1.
If we write 1.073600 to 6 s.f, the answer is 1.07360 because the 6th significant digit is 0 and the next digit is 0 also. So we round down.
If we write 1.073600 to 1 s.f, the answer is 1.

e) 263000 (2 s.f)
The 0 in this whole number are called trailing zeros and they insignificant. The 2nd significant digit is 6 and the next digit is 3. We round down and the answer is 260000.
If we write 263000 to 1 s.f, the answer is 300000 because the 1st significant digit is 2 and the next digit is 6. So we round up.

SCIENTIFIC NOTATION

Scientific notation (also referred to as "standard form" or "standard index form") is a way of writing numbers that are too big or too small to be conveniently written in decimal form. In scientific notation all numbers are written in the form $a \times 10^b$ (*a* times ten raised to the power of *b*), where the exponent *b* is an integer, and the coefficient *a* is any real number $\left(1 \le |a| < 10\right)$, called the mantissa. Source (http://en.wikipedia.org/wiki/Scientific_notation)

For large numbers, *b* will be positive whilst for small numbers, *b* will be negative.

Example 1: Write the following in standard form:

1) 163000
a = 1.63 because $1 < 1.63 < 10$ but not 16.3 or 163. b = 5 because in the number 163000, the decimal point is at end and strictly speaking we should write is as 163000. . So we have to move the decimal point 5 times to the left to obtain 1.63. Therefore the answer is 1.63×10^5
2) 2057400000
The answer is 2.0574×10^9. a = 2.0574 and we have to move the decimal point 9 times to the left to obtain 2.0574 which lies between 1 and 10.
3) 202.00587
The answer is 2.0200587×10^7. We have to move the decimal point 7 times to the left to obtain 2.0200587 which lies 1 and 10.
If we write 202.00587 to 3 s.f in standard form, the answer is 2.02×10^7.
4) 0.00000045689
The answer is 4.5689×10^{-7}. In this case, we have to move the decimal point to the right 7 times to obtain 4.5687 which lies between 1 and 10.
5) -0.00000000000000000000000000007
The answer is -7×10^{-29} because we have to move the decimal point 29 times to the right to obtain $|-7|$ which lies between 1 and 10.

Example 2: Write down the following numbers in full

1) 1.73×10^5
Since the power of 10 is positive, we move the decimal point 5 places to the right. Write 0 in the empty spaces to obtain 173000
2) 5.5×10^{-6}
This time the power of 10 is negative. So we move the decimal point 6 places to the left and the answer is 0.0000055.
3) 6.32^{06} displayed on a calculator
On a calculator 6.32×10^6 is written as above. Therefore, $6.32^{06} = 6320000$ in full.

Using Calculator

If you want your calculator to give all answers to a certain number of decimal place, use 'Fix' mode. When you shift to 'Fix' mode (SHIFT+Mode+6, depends on model of calculator), please specify the number of decimal places (0 ~ 9) that you want your answers to be displayed.

Similarly, if you go in 'Sci' mode (SHIFT+Mode+7), you can program your calculator to give your answers to a certain number of significant figures (0 ~ 9) and in standard form. To remove 'Fix' or 'Sci' mode, shift to 'Norm' mode (SHIFT+Mode+8)

To enter a number in standard form on your calculator, press 'Exp' or '$\times 10^x$'. For example to enter 6.5×10^{18}, type 6.5, then press 'Exp' or '$\times 10^x$' to enter power 18.